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Transient current through a quantum dot with two time-dependent barriers

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Abstract. We investigate the transient-current characteristics of a quantum dot coupled to two leads through two time-dependent barriers. A general formula for the time-dependent current $j(t)$ is derived by using the nonequilibrium-Green-function technique. Two particular cases, those of rectangular-pulse modulations and harmonic modulations imposed on the two barriers, are studied in detail. For the rectangular-pulse modulations, a turnstile effect is obtained if the full width of the resonant state (Γ) is much larger than the frequency (ω), i.e. $\Gamma \gg \omega$; and the special behaviour of the transient current $j(t)$ is attributed to the phase coherence due to the time variation of the barriers. For the harmonic modulations, we find that a new energy scale $\hbar\omega$ emerges if $\Gamma < \omega$, and the main resonant peak is split into two peaks.

1. Introduction

The study of quantum transport in mesoscopic systems has received considerable attention in the past decade, not only because of the fundamental physics, but also because of the great potential as regards the new generation of electronic and photonic devices. The transport properties studied so far have been mainly related to steady-state processes. Recently, time-dependent transport phenomena have begun to attract more and more attention. The essential feature of the mesoscopic physics is the phase coherence of the charge carriers. For the time-dependent processes, generally, the external time-dependent perturbation affects the phase coherence differently in different parts of the system [1]. A new energy scale $\hbar\omega$ in the time-dependent problem has been introduced. A multitude of new effects have been observed, for example: photon–electron pumps, the sideband effect, turnstiles, the ac response in resonant-tunnelling devices, and so on.

As regards the theoretical aspects, Tien and Gordon studied the effect of microwave radiation on superconducting tunnelling devices back in the early sixties [2]. Since then, different theoretical approaches have been developed, such as those of the time-dependent Schrödinger equation [3–5], the transfer Hamiltonian [6, 7], the Master equation [8, 9], the Wigner function [10], and the nonequilibrium-Green-function method [1, 11–13].

In this paper we consider a quantum dot coupled to two leads through two time-dependent barriers, and study the time-varying characteristics of the current through the system. Unlike in most of the previous work in which the time-dependent external fields

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are applied either to the leads or to the quantum dot [9, 14–16], here it is only to the barriers that the time-dependent external fields are applied. We assume that the changes of the heights of the two barriers do not affect the single-electron energies in the two leads and the dot, i.e. that all of the single-electron energies are still time independent. It should be mentioned that when time-dependent external fields are applied to the leads or to the dot, the phase of the electron wave-function will change from $\exp[-\mathrm{i}\varepsilon(t-t')]$ to $\exp[-\mathrm{i}\int_{t'}^{t}\mathrm{d}t_1\ \varepsilon(t_1)]$ [13]. However, when time-dependent external fields are applied only to the barriers, we find in this work that two main differences occur: (1) when the electron tunnels through the barrier, the phase changes from a time-independent value to a time-dependent one; and (2) the time variation of the barriers affects the tunnelling probability, destroys the steady state established originally, and produces an extra phase difference between the states on either side of the barrier.

It has become possible to modulate the heights of barriers experimentally [17–19], and an interesting effect—the turnstile effect—has been observed to occur when the barriers change periodically [17, 19].

In this work we first use the nonequilibrium-Green-function method and the transient transport theory of references [13] and [1] to derive a general formula for the time-dependent current $j(t)$. Then we study two particular cases: (1) a rectangular-pulse modulation is applied to each barrier, but the two modulations are out of phase (with a phase difference of π)—the turnstile effect is obtained if $\Gamma \gg \omega$, and a special behaviour of the transient current $j(t)$ emerges which can be explained as a phase coherence effect due to the time variation of the barriers; (2) instead of the rectangular-pulse modulations, two harmonic modulations with a phase difference of π are applied to the barriers. We find that a new energy scale $\hbar\omega$ emerges if $\Gamma < \omega$, and the main resonant peak splits into only two peaks, and not a set of peaks as for the usual sideband effect case [9].

The outline of this paper is as follows. In section 2, the model is presented and the Keldysh nonequilibrium Green function is used to derive the time-dependent current formulae. In section 3, we study the case with rectangular-pulse modulations. In section 4. the case with harmonic modulations is discussed. A brief summary is presented in section 5.

2. The model and formulation

The mesoscopic system under consideration is a quantum dot coupled to two leads through two time-dependent barriers. This system can be described by the following Hamiltonian:

$$H(t) = H_{lead} + H_{dot} + H_T(t)$$

where

$$\begin{aligned} H_{lead} &= \sum_{k\in L} \varepsilon_k a_k^\dagger a_k + \sum_{p\in R} \varepsilon_p b_p^\dagger b_p \\ H_{dot} &= \varepsilon_0 c^\dagger c \\ H_T(t) &= \sum_k L_k(t) a_k^\dagger c + \sum_p R_p(t) b_p^\dagger c + \mathrm{HC.} \end{aligned} \tag{1}$$

H_{lead} models noninteracting electrons in the leads, and $a_k^\dagger$ (a_k) and $b_p^\dagger$ (b_p) are the creation (annihilation) operators of the electron in the left-hand and right-hand lead, respectively. H_{dot} describes the quantum dot. For simplicity, we only consider a single state in the quantum dot, and neglect the intra-dot electron–electron Coulomb interaction. The tunnelling part is denoted by $H_T(t)$, which is the only part with time-dependent behaviour.

ε_k, ε_p, and ε_0 are the single-electron energies, corresponding to electrons in the left-hand lead, right-hand lead, and the dot, respectively. We assume that these energies are not affected by the time-dependent external fields applied to the left-hand and right-hand barriers.

In the following we derive the general time-dependent current formula $j(t)$ by using the nonequilibrium-Green-function technique and the theory by Wingreen, Jauho and Meir given in references [13] and [1]. The current tunnelling from the left-hand lead to the quantum dot can be calculated from the evolution of the occupation number operator $N_L = \sum_k a_k^\dagger a_k$ for the left-hand lead. One readily finds (in units in which $\hbar = 1$)

$$j_L(t) = -e\langle \dot{N}_L \rangle = -\mathrm{i}e\langle [H, N_L] \rangle = 2e\,\mathrm{Re}\sum_k L_k(t) G_{0k}^<(t,t). \tag{2}$$

Here we have defined the Green function $G_{0k}^<(t,t')$ as $G_{0k}^<(t,t') \equiv \mathrm{i}\langle a_k^\dagger(t')c(t)\rangle$. With the help of the Dyson equation, it can be written as

$$G_{0k}^<(t,t') = \int \mathrm{d}t_1\; L_k^*(t_1)\left[G_{00}^r(t,t_1)g_k^<(t_1,t') + G_{00}^<(t,t_1)g_k^a(t_1,t')\right] \tag{3}$$

where $G_{00}^r(t,t_1) \equiv -\mathrm{i}\theta(t-t_1)\langle\{c(t), c^\dagger(t_1)\}\rangle$, $G_{00}^<(t,t_1) \equiv \mathrm{i}\langle c^\dagger(t_1)c(t)\rangle$, and $g_k^<$ and g_k^a are the exact Green functions of the electron in the left-hand lead without coupling between the leads and the dot. Substituting the expression for $G_{0k}^<(t,t)$ into equation (2), the discrete sum over k in $\sum_k$ can be changed into an integral with the help of the density of states in the left-hand lead, $\int \mathrm{d}\varepsilon\; \rho_L(\varepsilon)$. Then the current $j_L(t)$ becomes

$$j_L(t) = -2e\int_{-\infty}^{t} \mathrm{d}t_1 \int \frac{\mathrm{d}\varepsilon}{2\pi}\;\mathrm{Im}\left\{\mathrm{e}^{-\mathrm{i}\varepsilon(t_1-t)}\Gamma^L(\varepsilon,t_1,t)\left[G_{00}^<(t,t_1) + f_L(\varepsilon)G_{00}^r(t,t_1)\right]\right\} \tag{4}$$

where $\Gamma^L(\varepsilon,t_1,t) \equiv 2\pi\rho_L(\varepsilon)L(\varepsilon,t)L^*(\varepsilon,t_1)$, and $f_L(\varepsilon)$ is the Fermi distribution function of electrons in the left-hand lead.

In the following we assume that: (1) one can factorize the energy and the time dependence of the tunnelling coupling as $L(\varepsilon,t) = L(t)V_L(\varepsilon)$ and $R(\varepsilon,t) = R(t)V_R(\varepsilon)$; and (2) the wide-band limit can be used in the calculation, which means that the energy level width of the quantum dot due to the tunnelling between the dot and the two leads can be taken as an energy-independent constant, i.e. $\Gamma^L(\varepsilon) = 2\pi\rho_L(\varepsilon)V_L(\varepsilon)V_L^*(\varepsilon)$ and $\Gamma^R(\varepsilon) = 2\pi\rho_R(\varepsilon)V_R(\varepsilon)V_R^*(\varepsilon)$. Then the formula for the current $j_L(t)$ can be reduced to

$$j_L(t) = -e\{\Gamma^L(t)N(t) + B(t)\} \tag{5}$$

where $\Gamma^\alpha(t) \equiv \Gamma^\alpha(t,t) = \Gamma^\alpha|\alpha(t)|^2$, $\alpha = L, R$, $N(t) = \mathrm{Im}\,G_{00}^<(t,t)$ is the occupation of the electrons in the quantum dot, and $B(t)$ is defined as

$$\begin{aligned} B(t) &= \Gamma^L\,\mathrm{Im}\int \frac{\mathrm{d}\varepsilon}{\pi}\; f_L(\varepsilon)\int_{-\infty}^{t}\mathrm{d}t_1\; L^*(t_1)L(t)\mathrm{e}^{\mathrm{i}\varepsilon(t-t_1)}G_{00}^r(t,t_1) \\ &= -\,\Gamma^L\,\mathrm{Re}\int\frac{\mathrm{d}\varepsilon}{\pi}\; f_L(\varepsilon)\int_{-\infty}^{t}\mathrm{d}t_1\; L^*(t_1)L(t) \\ &\quad\times\;\exp\!\left(\mathrm{i}(\varepsilon-\varepsilon_0)(t-t_1) - \int_{t_1}^{t}\frac{\mathrm{d}t_2}{2}\;[\Gamma^L(t_2)+\Gamma^R(t_2)]\right). \end{aligned} \tag{6}$$

From the Keldysh equation for $G_{00}^<$ which reads

$$G_{00}^<(t,t') = \int \mathrm{d}t_1 \int \mathrm{d}t_2\; G_{00}^r(t,t_1)\Sigma^<(t_1,t_2)G_{00}^a(t_2,t') \tag{7}$$

one finds

$$N(t) = \Gamma^L\int\frac{\mathrm{d}\varepsilon}{2\pi}\; f_L|A_L(\varepsilon,t)|^2 + \Gamma^R\int\frac{\mathrm{d}\varepsilon}{2\pi}\; f_R|A_R(\varepsilon,t)|^2 \tag{8}$$

where $f_R(\varepsilon)$ is the Fermi distribution function of electrons in the right-hand lead, and $A_\alpha(\varepsilon, t)$ is a compact notational form, defined as

$$A_\alpha(\varepsilon, t) = -\mathrm{i} \int_{-\infty}^{t} \mathrm{d}t_1 \ \alpha(t_1) \exp\left\{ \mathrm{i}(\varepsilon - \varepsilon_0)(t - t_1) - \int_{t_1}^{t} \frac{\mathrm{d}t_2}{2} \ \left[\Gamma^L(t_2) + \Gamma^R(t_2)\right] \right\} \tag{9}$$

in which $\alpha = L, R$. From equations (5), (6), (8), and (9), we can evaluate the time-dependent current $j_L(t)$ for the two barriers modulated by arbitrary time-dependent external fields. In the following we shall study two particular cases: rectangular-pulse modulations and harmonic modulations.

3. The response to rectangular-pulse modulations

3.1. Basic formulae

In this case a periodic rectangular-pulse external field, with period T, is applied to each barrier, but the two fields are out of phase (with a phase difference of π)—$L(t)$ and $R(t)$ are as follows:

$$\begin{cases} L(t) = 1, R(t) = 0 & 0 \leqslant t < T/2 \\ L(t) = 0, R(t) = 1 & T/2 \leqslant t < T. \end{cases}$$

Hence $\Gamma^L(t), \Gamma^R(t)$ can be found:

$$\begin{cases} \Gamma^L(t) = \Gamma^L, \Gamma^R(t) = 0 & 0 \leqslant t < T/2 \\ \Gamma^L(t) = 0, \Gamma^R(t) = \Gamma^R & T/2 \leqslant t < T. \end{cases} \tag{10}$$

In the first half-cycle, $\Gamma^R(t) = 0$, i.e. the height of the right-hand barrier is $+\infty$; hence the quantum dot only couples to the left-hand lead. Conversely, in the second half-cycle, the height of the left-hand barrier is $+\infty$; hence the quantum dot only couples to the right-hand lead. At the times $t = T/2$ and $t = T$, the left-hand and right-hand barriers vary simultaneously.

For simplicity we consider that $\Gamma_L = \Gamma_R \equiv \Gamma/2$, i.e. the left-hand and the right-hand barriers are symmetric in the absence of external fields, and the temperature is taken to be zero ($\mathcal{T} = 0$ K). Since the temperature in the experiments is usually low, say 50 mK [18–20], $k_B\mathcal{T}/\hbar\omega \approx 0.1$ (the quantum regime), so setting $\mathcal{T} = 0$ K is quite reasonable. Because $j_L(t)$ is a periodic function, we only need to calculate $j_L(t)$ for times in the range $0 \leqslant t < T$.

For $T/2 \leqslant t < T$, one finds $B(t) = 0$, so $j_L(t) = 0$.

For $0 \leqslant t < T/2$, by using equation (9) one can obtain $A_\alpha(\varepsilon, t)$ as

$$\begin{cases} A_L(\varepsilon, t) = \dfrac{1}{\varepsilon - \varepsilon_0 + \mathrm{i}\Gamma/2} \\ \qquad\qquad - \dfrac{\exp\left[-t(\mathrm{i}\varepsilon_0 - \mathrm{i}\varepsilon + \Gamma/2)\right]}{\varepsilon - \varepsilon_0 + \mathrm{i}\Gamma/2} \dfrac{1}{1 + \exp((-T/2)(\mathrm{i}\varepsilon_0 - \mathrm{i}\varepsilon + \Gamma/2))} \\ A_R(\varepsilon, t) = \dfrac{\exp\left[-t(\mathrm{i}\varepsilon_0 - \mathrm{i}\varepsilon + \Gamma/2)\right]}{\varepsilon - \varepsilon_0 + \mathrm{i}\Gamma/2} \dfrac{1}{1 + \exp(-(T/2)(\mathrm{i}\varepsilon_0 - \mathrm{i}\varepsilon + \Gamma/2))}. \end{cases} \tag{11}$$

Substituting the expressions for $A_L(\varepsilon t)$ and $A_R(\varepsilon t)$ into equation (8), $N(t)$ can be derived as

$$N(t) = \Gamma \int_{-\infty}^{\mu_L - \varepsilon_0} \frac{\mathrm{d}x}{2\pi} \left\{ \frac{1}{x^2 + \Gamma^2/4} + \frac{2\mathrm{e}^{-\Gamma t}}{x^2 + \Gamma^2/4} \frac{1}{|1 + \exp((T/2)(\mathrm{i}x - \Gamma/2))|^2} \right.$$

$$- 2\,\mathrm{Re}\left[\frac{\exp(t(\mathrm{i}x-\Gamma/2))}{x^2+\Gamma^2/4}\frac{1}{1+\exp((T/2)(\mathrm{i}x-\Gamma/2))}\right]\Bigg\}$$
$$- \Gamma\int_{\mu_R-\varepsilon_0}^{\mu_L-\varepsilon_0}\frac{\mathrm{d}x}{2\pi}\;\frac{\mathrm{e}^{-\Gamma t}}{x^2+\Gamma^2/4}\frac{1}{|1+\exp((T/2)(\mathrm{i}x-\Gamma/2))|^2}. \qquad (12)$$

Since $j_L(t)-j_R(t)=e\dot{N}(t)$, and for $0\leqslant t<T/2$ we have $j_R(t)=0$, then $j_L(t)=e\dot{N}(t)$. From equation (12), the time-dependent current $j_L(t)$ can be obtained straightforwardly:

$$j_L(t) = -e\Gamma\int_{-\infty}^{\mu_L-\varepsilon_0}\frac{\mathrm{d}x}{2\pi}\;\Bigg\{\frac{2\Gamma\mathrm{e}^{-\Gamma t}}{x^2+\Gamma^2/4}\frac{1}{|1+\exp((T/2)(\mathrm{i}x-\Gamma/2))|^2}$$
$$- \mathrm{Re}\left[\frac{(\Gamma-2\mathrm{i}x)\exp(t(\mathrm{i}x-\Gamma/2))}{(x^2+\Gamma^2/4)(1+\exp((T/2)(\mathrm{i}x-\Gamma/2)))}\right]\Bigg\}$$
$$+ e\Gamma^2\mathrm{e}^{-\Gamma t}\int_{\mu_R-\varepsilon_0}^{\mu_L-\varepsilon_0}\frac{\mathrm{d}x}{2\pi}\;\frac{1}{(x^2+\Gamma^2/4)}\frac{1}{|1+\exp((T/2)(\mathrm{i}x-\Gamma/2))|^2}. \qquad (13)$$

In the following two subsections we shall discuss the time-dependent characteristics of the current $j(t)$ for two specific conditions: $\mu_L=\mu_R$ and $\mu_L>\varepsilon_0>\mu_R$.

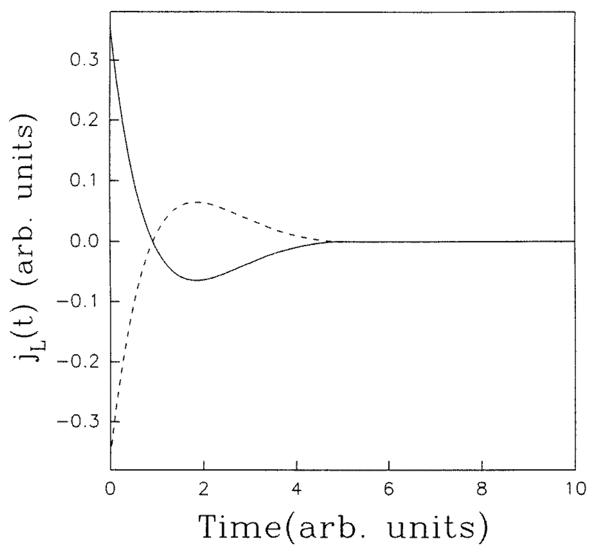

Figure 1. $j_L(t)$ for rectangular-pulse modulations of the two barriers, where $\Gamma=1$, $T=10/\Gamma$, and $\mu_L=\mu_R=0$. The broken and solid curves correspond to $\varepsilon_0=-\Gamma$ and $\varepsilon_0=\Gamma$, respectively.

3.2. The symmetric case: $\mu_L=\mu_R$

When $\mu_L=\mu_R\equiv\mu$, the second term of equation (13) is zero. The current $j_L(t)$ becomes

$$j_L(t) = -e\Gamma\int_{0}^{\mu-\varepsilon_0}\frac{\mathrm{d}x}{2\pi}\;\Bigg\{\frac{2\Gamma\mathrm{e}^{-\Gamma t}}{(x^2+\Gamma^2/4)|1+\exp((T/2)(\mathrm{i}x-\Gamma/2))|^2}$$
$$- \mathrm{Re}\left[\frac{(\Gamma-2\mathrm{i}x)\exp(t(\mathrm{i}x-\Gamma/2))}{(x^2+\Gamma^2/4)(1+\exp((T/2)(\mathrm{i}x-\Gamma/2)))}\right]\Bigg\}. \qquad (14)$$

In equation (14) we have already changed the lower limit of the integral from $-\infty$ to 0. In fact, by using the residual theorem one can prove that the integral from $-\infty$ to 0 is zero. The characteristics of $j_L(t)$ versus t are shown in figure 1: $j_L(t) \equiv 0$ (for $\mu = \varepsilon_0$), and $j_L(t) \neq 0$ (for $\mu \neq \varepsilon_0$). Considering $\mu > \varepsilon_0$ and during one period, the current $j(t)$ first flows from the quantum dot to the left-hand lead, and then flows in the opposite direction. Although the average current $\langle j_L(t) \rangle$ is zero, $j_L(t)$ is not even: $\mu_L = \mu_R \neq \varepsilon_0$. In fact the time variation of the barriers will affect the phase of the electron wave-function differently on the two sides of the barriers, cause interference, and make $j_L(t) \not\equiv 0$. In order to illustrate this rather complex phase change, we consider the following simple example.

The example is of the quantum dot coupled only to one lead (say, the left-hand lead), with $f(\varepsilon) = \delta(0)$ (i.e. only one electron occupies the $\varepsilon = 0$ energy level in the lead), and we set the height change of the left-hand barrier as follows: for $t < 0$, the barrier is fixed at a finite height ($\Gamma \neq 0$); for $0 \leqslant t < T$, the barrier is maintained at $+\infty$ ($\Gamma = 0$); at $t = T$, the barrier is reduced to the original height again, and maintains the height unchanged. Then, when $t < 0$, the electron tunnelling between the lead and the dot reaches a steady state with the occupation of the dot constant, and the current $j(t) = e\dot{N}(t) = 0$. When $0 \leqslant t < T$, the dot and the lead are completely separated, so $N(t)$ is unchanged and the current $j(t) = 0$. But during this time interval, the change of phase of the electron wave-function is different in the dot and in the lead, taking the values $\mathrm{i}\varepsilon_0 t$ and $\mathrm{i}\varepsilon t$, respectively. So at $t = T$, an extra phase difference $\mathrm{i}(\varepsilon_0 - \varepsilon)T$ will emerge, which affects the current $j(t)$ for $t > T$. In the meantime, when $t \geqslant T$, the quantum dot couples to the lead again. Due to the extra phase difference mentioned above, a coherence effect will occur in the tunnelling process, which breaks down the steady state established for the dot and the lead, and causes a nonzero current $j(t)$ as shown in figure 2.

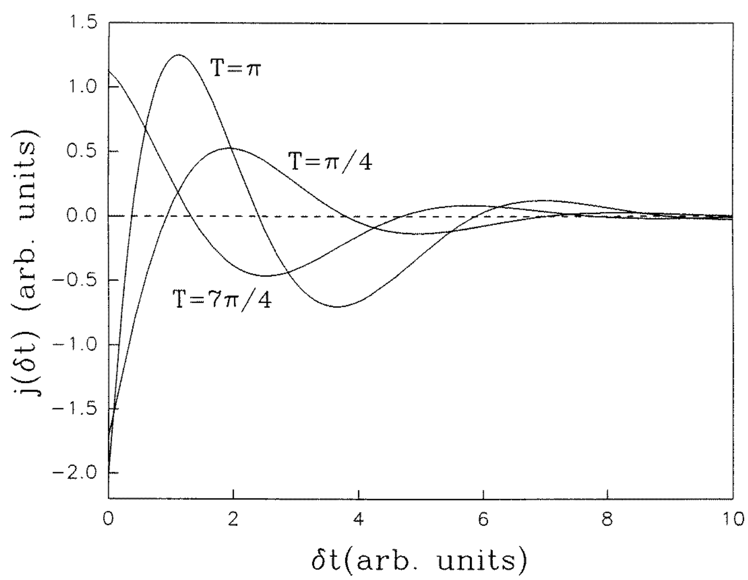

Figure 2. $j(\delta t)$ versus δt for a special system consisting of a dot coupled to only one lead, where $\delta t = t - T$, $\varepsilon_0 = 1$, $\varepsilon = 0$, and $\Gamma = 1$. The curves plotted correspond to $T = \pi/4, \pi$, and $7\pi/4$.

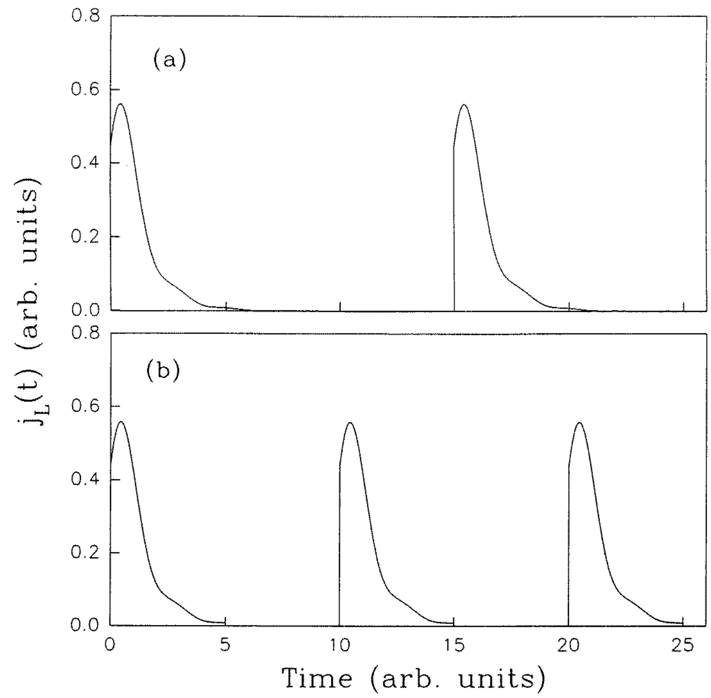

Figure 3. The characteristics of $j_L(t)$, displaying the turnstile effect, where $\mu_L = 3\Gamma$, $\mu_R = -3\Gamma$, and $\varepsilon_0 = 0$. (a) $T = 15/\Gamma$; (b) $T = 10/\Gamma$.

3.3. The turnstile effect

For $\mu_L > \varepsilon_0 > \mu_R$ and $\Gamma \gg \omega \equiv 2\pi/T$, the dot is coupled to the left-hand lead and separated from the right-hand lead in the first half-period. Since $\mu_L > \varepsilon_0$, the electron can tunnel from the left-hand lead to the dot. The situation will be completely reversed in the second half-period. Because $\Gamma \gg \omega$, there is only one electron passing through the quantum dot in every period, i.e. the turnstile effect occurs. This time-dependent feature of the current $j_L(t)$ is shown in figure 3.

From equation (13), one can easily calculate the average current $\langle j_L(t)\rangle$. Notice that the average of the first term in equation (13) is zero; one has

$$\langle j_L(t)\rangle = \frac{1}{T}\int_0^T j_L(t)\,\mathrm{d}t = -\frac{e\Gamma}{T}\left[\mathrm{e}^{-\Gamma T/2} - 1\right]\int_{\mu_R-\varepsilon_0}^{\mu_L-\varepsilon_0}\frac{\mathrm{d}x}{2\pi}\,\frac{1}{(x^2+\Gamma^2/4)|1+\exp((T/2)(\mathrm{i}x-\Gamma/2))|^2}. \tag{15}$$

For $\omega = 2\pi/T \ll \Gamma$, equation (15) is approximately equal to

$$\langle j_L(t)\rangle = \frac{e}{T\pi}\arctan\frac{2x}{\Gamma}\bigg|_{\mu_R-\varepsilon_0}^{\mu_L-\varepsilon_0}. \tag{16}$$

If $\mu_L - \varepsilon_0$ and $\varepsilon_0 - \mu_R$ are large enough compared to Γ, the average current $\langle j_L(t)\rangle$ reduces to

$$\langle j_L(t)\rangle = \frac{e}{T} = e\omega/2\pi \tag{17}$$

i.e. $\langle j_L(t)\rangle$ depends only on the frequency ω of the external fields, and exactly one electron passes through the quantum dot every period.

If T is small enough (or the frequency high enough), the two barriers will vary quickly. Finally, when $1/T \sim \Gamma$, the electron cannot tunnel—it does not have enough time for an electron to tunnel through the quantum dot in a period of time—so the average current $\langle j_L(t)\rangle$ will deviate downward from $e\omega/2\pi$.

Experimental results showing the turnstile effect were reported in references [17] and [19]; it was manifested as a set of plateaux in the I–v curves. Here we present our calculated time-dependent behaviour of $j(t)$ versus t (see figure 3), and the average current $\langle j(t)\rangle$ versus $1/T$ (see equation (16) and equation (17)). For $I = \langle j(t)\rangle$ versus v, only two plateaux at $I = \pm e\omega/2\pi$ are obtained, because only one single-electron state in the dot is considered.

4. The response to harmonic modulations

Now let us consider the case in which the external fields applied to the two barriers are harmonic and out of phase (with a phase difference of π). Then $\alpha(t)$ and $\Gamma^\alpha(t)$ are given by

$$\begin{cases} L(t) = \cos\omega t \\ R(t) = \sin\omega t \end{cases} \quad \text{and} \quad \begin{cases} \Gamma^L(t) = \Gamma^L \cos^2\omega t \\ \Gamma^R(t) = \Gamma^R \sin^2\omega t. \end{cases} \tag{18}$$

In the following we assume that the amplitudes of the left-hand and right-hand barriers are equal ($\Gamma^L = \Gamma^R = \Gamma/2$) in the absence of external fields, and the temperature is taken to be zero ($T = 0$ K). By using equations (5)–(9), one can get the time-dependent current $j_L(t)$. From equations (8), (9) and (18), one finds

$$\begin{aligned} N(t) = \Gamma \int_{-\infty}^{\mu_R-\varepsilon_0} \frac{\mathrm{d}x}{2\pi} \, \frac{x^2+\Gamma^2/4+\omega^2}{a} \\ + \Gamma \int_{\mu_R-\varepsilon_0}^{\mu_L-\varepsilon_0} \frac{\mathrm{d}x}{2\pi} \, \frac{x^2\cos^2\omega t + ((\Gamma/2)\cos\omega t + \omega \sin\omega t)^2}{a} \end{aligned} \tag{19}$$

in which a denotes $a = [(x+\omega)^2 + \Gamma^2/4][(x-\omega)^2+\Gamma^2/4]$. By using equation (6), we obtain

$$\begin{aligned} B(t) = -\int_{-\infty}^{\mu_L-\varepsilon_0} \frac{\mathrm{d}x}{\pi} \, \Gamma\cos\omega t \\ \times \frac{(\Gamma/2)(\Gamma^2/4+x^2+\omega^2)\cos\omega t + \omega(\Gamma^2/4 - x^2+\omega^2)\sin\omega t}{a}. \end{aligned} \tag{20}$$

Substituting equations (19) and (20) into equation (5), the time-dependent current $j_L(t)$ can be written as

$$\begin{aligned} j_L(t) = e \int_{\mu_R-\varepsilon_0}^{\mu_L-\varepsilon_0} \frac{\mathrm{d}x}{2\pi} \, \Gamma^2\cos^2\omega t \, \frac{(\Gamma^2/4+x^2)\sin^2\omega t + \omega^2\cos^2\omega t - \Gamma\omega\cos\omega t\sin\omega t}{a} \\ + e\int_0^{\mu_L-\varepsilon_0} \frac{\mathrm{d}x}{2\pi} \, \frac{\Gamma^2/4 - x^2+\omega^2}{a}\,\Gamma\omega\sin 2\omega t. \end{aligned} \tag{21}$$

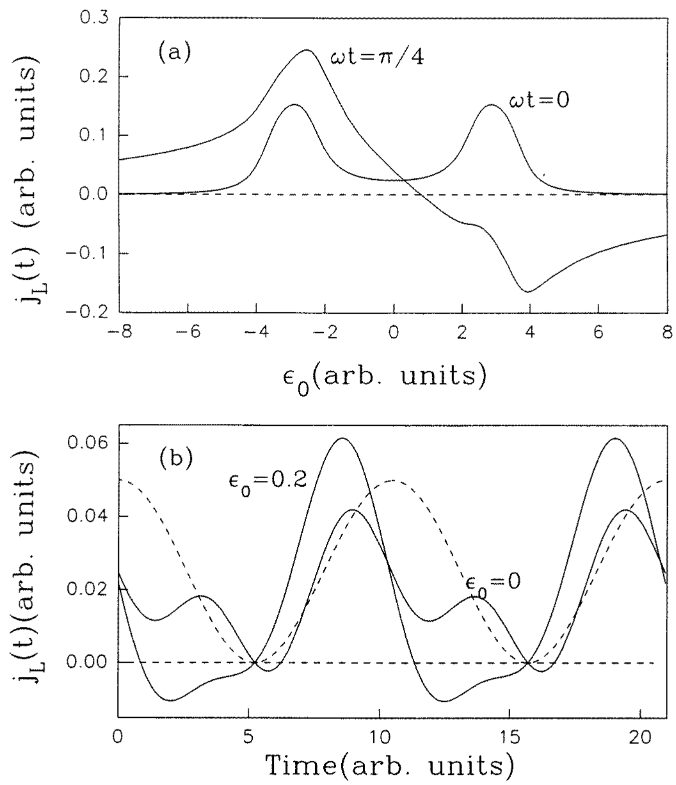

Figure 4. $j_L(t)$ for harmonic modulations of the two barriers. (a) $j_L(t)$ versus ε_0 at different times ($\omega t = 0$ and $\omega t = \pi/4$), for $\mu_L = -\mu_R = 0.7\Gamma$, and $\omega = 3\Gamma$. (b) $j_L(t)$ versus t for different values of ε_0, for $\mu_L = -\mu_R = 0.1\Gamma$, and $\omega = 0.3\Gamma$: the two solid curves correspond to $\varepsilon_0 = 0$ and $\varepsilon_0 = 0.2\Gamma$, and the broken curve shows $\Gamma^L(t)$ versus t.

Figure 4(a) shows $j_L(t)$ versus ε_0 at different times. The curves for $j_L(t)$ versus t for different values of ε_0 are given in figure 4(b).

Now we consider two special examples.

(1) $\mu_L = \mu_R$, or the source–drain voltage $v = \mu_L - \mu_R = 0$. Then equation (21) reduces to

$$j_L(t) = \frac{e\Gamma}{8\pi} \sin\omega t \ln \frac{\Gamma^2/4 + (\mu_L - \varepsilon_0 + \omega)^2}{\Gamma^2/4 + (\mu_L - \varepsilon_0 - \omega)^2} \tag{22}$$

and

$$N(t) = \Gamma \int_{-\infty}^{\mu_L - \varepsilon_0} \frac{\mathrm{d}x}{2\pi}\ (x^2 + \Gamma^2/4 + \omega^2)/a$$

is time independent. Since the change of the barriers affects the phase coherence differently in different parts of the system, $j_L(t) \neq 0$ (for $\mu_L = \mu_R \neq \varepsilon_0$). From equation (22) it is clear that when $\Gamma \ll \omega$, two peaks emerge at the locations of $\varepsilon_0 = \mu_L \pm \omega$, i.e. the photon is present and assists the tunnelling.

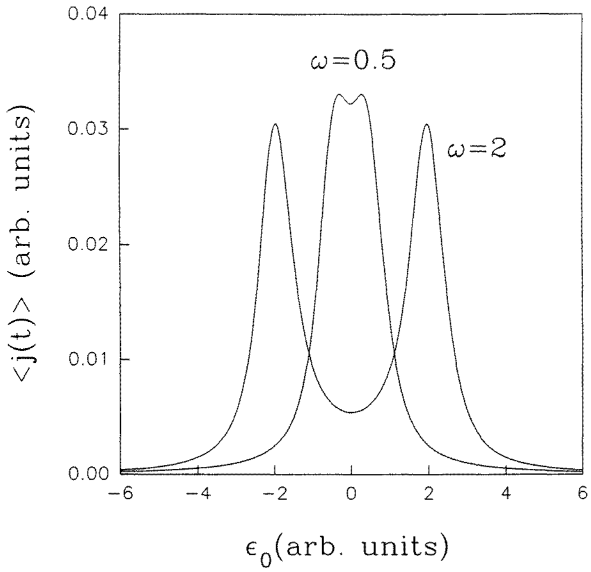

Figure 5. Average currents $\langle j(t)\rangle$ versus ε_0 for harmonic modulations of the two barriers, corresponding to $\omega = 2\Gamma$ and $\omega = 0.5\Gamma$, where $\mu_L = -\mu_R = 0.2\Gamma$ and $\Gamma = 1$.

(2) The average current $\langle j(t)\rangle$. From equation (21) we can derive the average current $\langle j(t)\rangle$:

$$\langle j(t)\rangle = \frac{e\Gamma^2}{8}\int_{\mu_R-\varepsilon_0}^{\mu_L-\varepsilon_0}\frac{\mathrm{d}x}{2\pi}\;\frac{x^2+3\omega^2+\Gamma^2/4}{a}. \tag{23}$$

When $\Gamma < \omega$, the main resonant peak is split into two peaks at $\pm\hbar\omega$ from the original main peak which has now disappeared (see figure 5). This is different from the situation studied in reference [9] in which two harmonic external fields are applied to the left-hand and right-hand leads symmetrically, leading to the main peak splitting into a set of peaks. In our situation only two peaks emerge. This can be explained by photon-assisted tunnelling, i.e. the electron can tunnel between the lead and the quantum dot by absorbing or emitting a photon. Since $L(t) = \cos\omega t = (\mathrm{e}^{\mathrm{i}\omega t} + \mathrm{e}^{-\mathrm{i}\omega t})/2$ and $R(t) = \sin\omega t = (\mathrm{e}^{\mathrm{i}\omega t} - \mathrm{e}^{-\mathrm{i}\omega t})/2\mathrm{i}$, the phase of the electron wave-function will vary between $\pm\mathrm{i}\omega t$ when the electron tunnels through the left-hand or right-hand barrier, leading to only two peaks at $\pm\omega$ away from the original main peak. On reducing ω, these two peaks will approach each other, and finally become one single peak—the original main peak.

5. Conclusion

In this paper, we have used the nonequilibrium-Green-function method to study the time-varying behaviour of electrons tunnelling through a quantum dot with two time-dependent barriers. The main result presented is the expression for the transient current $j(t)$ for some special cases. We find that the transient current $j(t)$ displays a diverse behaviour in different situations, exhibiting a turnstile effect, photon-assisted tunnelling, etc. Except the $\langle j(t)\rangle$ versus v turnstile effect, the results presented in this work have not been observed experimentally.

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